**Math Pac 2  
Owner's Manual Supplement**

**For the HP-71**

**Math Pac 2**

**Owner's Manual Supplement**

**For Use with the HP-71**

**July 2020**

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Web site: http://www.jeffcalc.hp41.eu/emu71/mathrom.html

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**Introducing the Math Pac 2**

The Math Pac 2 is an enhanced version of the 1984 Math Pac for the HP-71 Computer.  
The original Math Pac provided a set of powerful tools for solving a wide range of mathematical, scientific and engineering problems.

The Math Pac 2 adds several complements by integrating the best features from other computers/calculators such as the HP-75 Computer, the Series 80 Desktop Computers, the Series 200/300 HP BASIC, the HP-28C/S Advanced Scientific Calculators and the HP-42S RPN Scientific Calculator, to reach state-of-the-art levels in terms of mathematical tools in a single BASIC environment. You will never have to use an RPN/RPL calculator or a bulky HP desktop computer for special computing purposes 1.

The Math Pac 2 adds the following capabilities to your HP-71:

* Full set of the original Math Pac 1A capabilities.
* Additional real- and complex-valued functions.
* Additional array functions.
* Additional matrix operations.
* Improvements in several functions.
* New keywords for functional compatibility with other computers and calculators.

1. This should not be taken too seriously, RPN/RPL calculators are very good and the HP BASIC (also known as Rocky Mountain BASIC) is still a powerful language for technical and instrumentation applications.

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**How to Use This Manual**

This supplement assumes that you are familiar with the operation of the 1984 Math Pac 1A, which is not described again in this manual. Only the new capabilities and keywords provided in the Math Pac 2 are described here. Please refer to the 1984 Math Pac Owner's Manual for a reference to the original capabilities. However, the keyword index at the end of this manual summarizes all the capabilities of the Math Pac 2 including the original Math Pac capabilities.

Additional Information

Additional information can be found in the manuals for the HP-71, HP-75, Series 80, Series 200/300 BASIC and HP calculators, specifically:

* The HP-71 Math Pac Owner's Manual.
* The HP-75 Math Pac Owner's Manual.
* The Series 80 Matrix ROM Manual.
* The Series 200/300 BASIC Language Reference Manual.

Section 1

**Installing the Math 2 LEX File**

The Math 2 LEX file is installed from the Math 2 Distribution Pac.

**CAUTIONS**

* Don't install the Math 2 LEX together with the HP-71 Math ROM. This will definitively cause problems.
* When installing the Math 2 LEX for the first time, and if a Math ROM was previously installed, be sure to turn on the HP-71 AFTER removing the Math ROM, but BEFORE installing the Math Pac 2 (that is, with none of the Math ROM or Math 2 LEX present). This will guarantee that the new Math 2 LEX will be correctly initialized.  
  This step is not needed when later installing new versions of the Math 2 LEX.
* The Math Pac 2 is backward compatible with the Math ROM (old programs will run correctly on the Math Pac 2), but programs using the new Math Pac 2 capabilities should not be run with the previous Math ROM installed. See Appendix A for details.

Please refer to the Math 2 Distribution Pac documentation for instructions on how to install the Math 2 LEX file in your HP-71 or in an HP-71 emulator.

Section 2

**New Real- and Complex-Valued Functions**

Real-Valued Trigonometric Functions

The three functions described below are for HP-75 and Series 80 compatibility.

**SEC Secant**

SEC(X)

Where X is a real-valued numeric expression.  
The secant is defined by SEC(X)=1/COS(X)in the current angular setting.

Can be used in CALC mode.

**CSC Cosecant**

CSC(X)

Where X is a real-valued numeric expression.  
The cosecant is defined by CSC(X)=1/SIN(X) in the current angular setting.

Can be used in CALC mode.

**COT Cotangent**

COT(X)

Where X is a real-valued numeric expression.  
The cotangent is defined by COT(X)=1/TAN(X) in the current angular setting.

Can be used in CALC mode.

Complex-Valued Inverse Hyperbolic Functions

The Math Pac 2 adds support for complex arguments in the inverse hyperbolic functions.

**ASINH Inverse Hyperbolic Sine**

ASINH(Z)

Where Z is a complex-valued numeric expression.  
Returns the complex principal value of the inverse hyperbolic sine of Z.  
The complex inverse hyperbolic sine is defined by:  
 ASINH(Z)=LOG(Z+SQRT(1+Z^2)) 1

Can be used in CALC mode.

**ACOSH Inverse Hyperbolic Cosine**

ACOSH(Z)

Where Z is a complex-valued numeric expression.  
Returns the complex principal value of the inverse hyperbolic cosine of Z.  
The complex inverse hyperbolic cosine is defined by:  
 ACOSH(Z)= SGN(IMPT(Z))\*LOG(Z+(0,1)\*SQRT(1-Z^2)) 1

Can be used in CALC mode.

**ATANH Inverse Hyperbolic Tangent**

ATANH(Z)

Where Z is a complex-valued numeric expression, Z≠(±1,0)  
Returns the complex principal value of the inverse hyperbolic tangent of Z.  
The complex inverse tangent is defined by:  
 ATANH(Z)=LOG((1+Z)/(1-Z))/2 1

Can be used in CALC mode.

1. The Math Pac 2 internally uses special algorithms to provide accurate results over the whole argument range.

Complex-Valued Inverse Trigonometric Functions

The Math Pac 2 adds support for complex arguments in the inverse trigonometric functions. ASN/ACS/ATN are for compatibility with the Series 200/300 HP BASIC.

**ASIN/ASN Inverse Sine**

ASIN(Z) or ASN(Z)

Where Z is a complex-valued numeric expression.  
Returns the complex principal value of the inverse sine of Z.  
The complex inverse sine is defined by:  
ASIN(Z)=(0,-1)\*ASINH((0,1)\*Z) 1

Can be used in CALC mode.

**ACOS/ACS Inverse Cosine**

ACOS(Z) or ACS(Z)

Where Z is a complex-valued numeric expression.  
Returns the complex principal value of the inverse cosine of Z.  
The complex inverse cosine is defined by:  
ACOS(Z)= SGN(IMPT(Z))\*(0,-1)\*ACOSH(Z) 1

Can be used in CALC mode.

**ATAN/ATN Inverse Tangent**

ATAN(Z) or ATN(Z)

Where Z is a complex-valued numeric expression, Z≠(0,±1)  
Returns the complex principal value of the inverse tangent of Z.  
The complex inverse tangent is defined by:  
ATAN(Z)=(0,-1)\*ATANH((0,1)\*Z) 1, 2

Can be used in CALC mode.

1. The Math Pac 2 internally uses special algorithms to provide accurate results over the whole argument range.  
2. A warning or error message "ATANH=Inf" is generated for Z=(0,1)and Z=(0,-1).

Complex-Valued Base-10 Logarithm Function

The Math Pac 2 adds support for complex arguments in the base-10 logarithm function for compatibility with Series 200/300 HP BASIC LGT function and the RPN/RPL calculators LOG function.

**LGT/LOG10 Base-10 Logarithm**

LGT(Z)or LOG10(Z)

Where Z is a complex-valued numeric expression, Z≠(0,0)  
The complex base-10 logarithm is defined by LGT(Z)=LOG(Z)/LOG(10)

Can be used in CALC mode.

Additional Information 1

For some inverse functions, the definitions of the principal branches are not universally agreed upon. In this discussion, uppercase letters will denote a single-valued inverse function (like   
COS-1 *z*) to distinguish from its multivalued inverse (cos-1 *z*).

The branches used by the Math Pac were chosen such that they are analytic in the regions where their real-valued counterparts are defined; that is, the branch cut occurs where the real-valued inverse is undefined. In addition, most of the important symmetries are preserved. For example:

for all *z*.

The illustrations that follow show the principal branches of the new inverse functions that the Math Pac 2 calculates.



1. The information in this section is adapted from the HP-75 Math Pac Owner's Manual.







The principal branches in these four graphs above are obtained from the equations shown, but don't necessarily use the principal branches of ln(z) and sqr(z).

The remaining inverse functions may be determined from the illustrations above and the following equations:

To determine all values of the inverse functions, use the expressions below to derive these values from the principal values calculated by the Math Pac 2. In these expressions, *k*=0, ±1, ±2, and so on.

Section 3

**New Scalar-Valued Array Functions**

Array Element Sum Functions

The SUM function provides compatibility with the HP-75, Series 80 and 200/300 HP BASIC.  
The ABSUM function is for HP-75 and Series 80 compatibility.

**SUM Array Element Sum**

SUM(A)

Where A is a real- or complex-type array.  
Returns the sum of the values of all elements in the array.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

**ABSUM Array Element Absolute Value Sum**

ABSUM(A)

Where A is a real-type array.  
Returns the sum of the absolute values of all elements in the array.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

Array Minimum/Maximum Element Functions

The four functions described below are for HP-75 and Series 80 compatibility.

**AMIN Array Element Minimum**

AMIN(A)

Where A is a real-type array.  
Returns the value of the minimum element in the array.  
Store the numbers of the row and column where the element is found for AROW and ACOL.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

**AMAX Array Element Maximum**

AMAX(A)

Where A is a real-type array.  
Returns the value of the maximum element in the array.  
Store the numbers of the row and column where the element is found for AROW and ACOL.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

**MINAB Array Element Minimum Absolute Value**

MINAB(A)

Where A is a real-type array.  
Returns the value of the smallest element (in absolute value) in the array.  
Store the numbers of the row and column where the element is found for AROW and ACOL.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

**MAXAB Array Element Maximum Absolute Value**

MAXAB(A)

Where A is a real-type array.  
Returns the value of the largest element (in absolute value) in the array.  
Store the numbers of the row and column where the element is found for AROW and ACOL.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

Array Row/Column Functions

The two functions described below are for Series 80 compatibility.

**AROW Array Row**

AROW

Returns the value of the row set by the last execution of one of the functions RNORM, AMIN, AMAX, MINAB or MAXAB.

Usable in CALC mode.

**ACOL Array Column**

ACOL

Returns the value of the column set by the last execution of one of the functions CNORM, AMIN, AMAX, MINAB or MAXAB.

Usable in CALC mode.

Additional Information

The AROW function combines the Series 80 RNORMROW/AMINROW/AMAXROW/MAXABROW functions in a single keyword. The ACOL function combines the CNORMCOL/AMINCOL/ AMAXCOL/MAXABCOL functions in a single keyword.

If more than one element is found in the AMIN/AMAX/MINAB/MAXAB functions, the element in the lowest-numbered row is chosen, and this number is returned as the value of the function AROW. If both such elements are in the same row, the element in the lowest-numbered column is chosen, and this number is returned as the value of the function ACOL.

The error "No Data" is generated if the AROW or ACOL function is executed before resp. the RNORM or CNORM function, and before any of the AMIN, AMAX, MINAB or MAXAB functions, since the last installation of the Math Pac 2.

Example 1

5 ! Example adapted from the HP-85 Matrix ROM Manual  
10 OPTION BASE 1  
20 DIM A(3,3),B(3,3),V1(3),V2(3)  
30 DATA -3,2,3,5,-3,5,2,5,-1  
40 DATA 2,1,3,1,4,2  
70 READ A(,)  
80 READ V1(),V2()  
90 MAT PRINT A;V1,V2  
100 PRINT ABSUM(A)  
110 PRINT AMAX(A)  
120 PRINT ACOL  
130 PRINT AROW  
140 PRINT AMIN(A)  
150 PRINT ACOL  
160 PRINT AROW  
170 PRINT CNORM(A)  
180 PRINT ACOL  
190 PRINT DET(A)  
200 MAT B=INV(A)  
210 PRINT DETL  
220 PRINT DOT(V1,V2)  
230 PRINT USING "2D.2D";FNORM(A)  
240 PRINT LBND(A,1)  
250 PRINT MAXAB(A)  
260 PRINT ACOL  
270 PRINT AROW  
280 PRINT RNORM(A)  
290 PRINT AROW  
300 PRINT SUM(A)  
310 N=2  
320 PRINT UBND(A,N)  
330 END

>RUN

-3 2 3 Array A  
 5 -3 5   
 2 5 -1

2 Vector V1  
 1   
 3

1. Except from the lines with the AROW/ACOL keywords, this program can also be run on the HP-75 with the Math Pac.

1 Vector V2  
 4   
 2

29 ABSUM(A): the sum of the absolute values of the elements in A.

5 AMAX(A): the value of the largest element in A.

1 ACOL: the lowest-numbered column containing AMAX(A).

2 AROW: the lowest-numbered row containing AMAX(A).

-3 AMIN(A): the value of the smallest element in A.

1 ACOL: the lowest-numbered column containing AMIN(A).

1 AROW: the lowest-numbered row containing AMIN(A).

10 CNORM(A) : the largest sum of the absolute values of the elements in each column of A.

1 ACOL: the lowest-numbered column with the largest sum of absolute values.

189 DET(A): Determinant of A.

189 DETL: Determinant of matrix A inverted in preceding MAT..INV statement.

12 DOT(V1,V2): the sum of the products of corresponding elements of V1 and V2.

10.54 FNORM(A): the square root of the sum of the squares of the elements in A.

1 LBND(A,1): the lower bound of the first subscript of A.

5 MAXAB(A): the largest absolute value of any element in A.

1 ACOL: the lowest-numbered column containing the element with largest absolute   
value.

2 AROW: the lowest-numbered row containing the element with largest absolute value.

13 RNORM(A): the largest sum of the absolute values of the elements in each row of A.

2 AROW: the lowest-numbered row with the largest sum of absolute values.

15 SUM(A): the sum of the elements in A.

3 UBND(A,2): the upper bound of the second subscript of A.

Complex Matrix Determinant

**DET/CDET Complex Determinant**

DET(A) or CDET(A)

Where A is a square real- or complex-type matrix.  
Returns the determinant of the matrix A.  
If A is real, then the result is real else the result is complex.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

Additional Information

The DET function is now extended to handle complex matrices.

CDET is equivalent to DET but doesn't accept the syntax CDET without operand.

Note that the DET without operand and the DETL functions don't provide the determinant of the last complex matrix used in the MAT..SYS, MAT..INV or MAT..INV\* statements. The reason is that the determinant is not computed in the process of inverting a complex matrix. The DET or CDET function has to be used to get the determinant of a complex matrix.

Alternate Inner Product

DOT2 is an alternate form of the inner (dot) product function, for compatibility with RPN/RPL calculators and Series 200/300 BASIC.

**DOT2 Alternate Inner (Dot) Product**

DOT2(A,B)

Where A and B are real- or complex-type arrays with the same number of elements.  
Returns A.B the inner product of A and B. If both A and B are real, then the result is real.  
If either A or B is complex, then the result is complex.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

Additional Information

For real vectors, the DOT2 function is equivalent to DOT.

For complex vectors, DOT2 differs from DOT in that the DOT2 inner product uses the ordinary complex product of corresponding vector elements, instead of using the complex conjugate of the elements of the first vector. This ensures compatibility with the HP-28S and HP-42S calculators and the Series 200/300 BASIC.

DOT2 also accepts matrices as arguments. The matrices A and B must have the same numbers of rows and columns. The inner product is computed on all corresponding elements of the matrices. This feature comes from the HP-28S and HP-42S calculators.

Section 4

**New Matrix Operations**

Cross Product Operation

The MAT..CROSS operation provides compatibility with the HP-75 and Series 80.

**CROSS Cross Product**

MAT A=CROSS(B,C)

Where A,B and C are vectors (1-dimension arrays).  
Both B and C must be real-type vectors and have 2 or 3 elements.

Implicitly redimensions A to have exactly three elements and assigns to A the values of the cross product B x C.

Not usable in CALC mode.

Additional Information

The operands of the CROSS statement can each have two or three elements. If missing, the third element is replaced by zero. If both operands have two elements only, then the elements 1 and 2 of the result are zero. The MAT A=CROSS(B,C) is then a quick way to  
compute the quantity B(1)\*C(2)-B(2)\*C(1) .

This feature comes from the HP-28S and HP-42S calculators.

Example

> OPTION BASE 1  
> DIM A(3),B(2),C(2)  
> MAT INPUT B,C  
B(1)? 2,7  
C(1)? 5,8  
> MAT A=CROSS(B,C)  
> DISP A(3)  
-19 This is 2\*8-7\*5

Row and Column Sum Operations

The MAT..RSUM and MAT..CSUM operations provide compatibility with the HP-75, the Series 80 and the Series 200/300 BASIC.

**RSUM Row Sum**

MAT A=RSUM(B)

Where A is a vector and B is a matrix.  
Matrix B may be either real or complex type.

Implicitly redimensions A to have as many elements as there are rows in B and assigns  
the sum of the values in each row of B to the corresponding element of A.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

**CSUM Column Sum**

MAT A=CSUM(B)

Where A is a vector and B is a matrix.  
Matrix B may be either real or complex type.

Implicitly redimensions A to have as many elements as there are columns in B and assigns  
the sum of the values in each column of B to the corresponding element of A.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

Additional Information

If the operand B is complex type, then A must be complex.  
If B is real, then A may be real or complex; if complex, all imaginary parts of all elements  
in A are set to zero.

MAT..CSUM can't be used to get the sum of the elements of a vector, use the array function SUM instead.

Matrix Division Operation

The MAT..INV()\* operation provides compatibility with the Series 80.

**INV \* Matrix Division**

MAT X=INV(A)\*B

Where A is a square matrix, X and B are both vectors or both matrices, and A and B are conformable for multiplication.   
Arrays A and B may be either real or complex type. If either A or B is complex, then X must be complex.  
If both A and B are real, then X may be real or complex; if complex, all imaginary parts of  
all elements in x are set to zero.

Implicitly redimensions X to be the same size as B and assigns to X the computed solution  
to the matrix equation A X=B.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

Additional Information

MAT X=INV(A)\*B directly computes the solution of the matrix equation A X=B instead of inverting A then multiplying by B. It is however not exactly equivalent to MAT X=SYS(A,B) in that it doesn't compute the residual correction that MAT..SYS does.

MAT X=INV(A)\*B uses less memory than MAT X=SYS(A,B) and is faster and generally more accurate than doing MAT A=INV(A) then MAT X=A\*B.

The MAT X=INV(A)\*B operation is the exact equivalent of the matrix division operation on the HP-28S and HP-42S calculators, hence the name "matrix division" for MAT..INV()\*.

The table below summarizes the three methods available to solve the matrix equation A X=B, together with their benefits. Each method can produce a slightly different result.

**System solving comparison**

Operations Features

MAT X=SYS(A,B) Best accuracy, more memory used.

MAT X=INV(A)\*B Less memory used, no residual correction.

MAT A=INV(A) @ MAT X=A\*B Matrix inversion done only once for solving   
 several systems.

Example

Solving the system = .

> OPTION BASE 1  
> DIM A(3,3),B(3),X(3)  
> MAT INPUT A,B  
A(1,1)? 69,58,95  
A(2,1)? 51,43,71  
A(3,1)? 32,55,54  
B(1)? 692,515,445

Using MAT..SYS :  
> MAT X=SYS(A,B)  
> MAT DISP X  
 2  
 3  
 4

Using MAT..INV()\* :  
> MAT X=INV(A)\*B  
> MAT DISP X  
 1.99999999732  
 2.99999999918  
 4.00000000242

Using MAT..INV and MAT..\* :  
> MAT A=INV(A)  
> MAT X=A\*B  
> MAT DISP X  
 2.00000000360  
 3.00000000053  
 3.99999999890

*LU* Decomposition

The MAT..LUFACT operations provides compatibility with the HP-75.

**LUFACT *LU* Decomposition**

MAT A=LUFACT(B)

Where B is a square real matrix, and A is a matrix.

Implicitly redimensions A to be the same size as B and assigns to A the values of the *LU* decomposition of B: the elements in A above the diagonal are assigned the value of the corresponding elements in U and the elements in A on or below the diagonal are assigned the value of the corresponding elements in L.

To halt operation, press [ATTN] twice.  
Not usable in CALC mode.

Example

> OPTION BASE 1  
> DIM A(3,3)  
> MAT INPUT A  
A(1,1)? 1,1,1  
A(2,1)? 1,0,2  
A(3,1)? 2,1,2  
> MAT A=LUFACT(A)  
> MAT DISP A;  
 2 .5 1  
 1 -.5 -2  
 1 .5 1

The L and U parts of the *LU* decomposition of A are respectively:

and ,

so L = and U =

Additional Information 1

The Math Pac *LU* decomposition factors a square matrix A into the matrix product LU, where

* L is a lower-triangular matrix - it has values of zero for all elements above the diagonal.
* U is a upper-triangular matrix - it has values of zero for all elements below the diagonal - with values of one for all elements on the diagonal.

For example, A = = = LU.

Some matrices can't be factored into LU form. For example, A = ≠ LU for *any* pair of lower- and upper-triangular matrices L and U. However, if rows are interchanged in the matrix to be decomposed, then *any* *non-singular* matrix can be so decomposed. Row interchanges in the matrix A can be represented by the matrix product PA for some permutation matrix P. Allowing for row interchanges, the *LU* decomposition can be represented by the equation  
PA = LU. So, for the above example

PA = = = = LU.

Row interchanges can also reduce rounding errors that can occur during the calculation of the decomposition.

The *LU* decomposition is returned in the form .  
For example, if the result of the MAT A=LUFACT(B) statement is

A = , then L = and U =

and PB = LU for some row interchange matrix P.

It is not necessary to store the diagonal elements of U in the result matrix since the value of each of these is equal to one. The row interchanges are also recoverable in many cases because, aside from row interchanges, the first column of L is the same as the first column of the matrix being decomposed.

For example, if B = and MAT A=LUFACT(B) is executed, then A = .

The fact that the first column of A is reversed from the first column of B indicates that the rows have been interchanged, so that

PB = = = = LU.

1. The information in this section is adapted from the HP-75 Math Pac Owner's Manual.

In many cases, the *LU* decomposition will be correct *even if the matrix is singular*. This can be checked by remultiplying the L and U matrices and comparing the result to the original matrix. This feature gives you the ability to find the *LU* decomposition of matrices that are not square.

For example, to find the *LU* decomposition of the 3 x 4 matrix ,

we will find the decomposition of instead.

From this decomposition, the *LU* decomposition of the original matrix is easily found.  
The keystroke sequence below illustrates the process (assuming OPTION BASE 1).

> DIM S(4,4)  
> MAT INPUT S  
S(1,1)? 1,2,3,4  
S(2,1)? 1,4,9,16  
S(3,1)? 1,8,27,64  
S(4,1)? 0,0,0,0  
> MAT S=LUFACT(S)  
> MAT DISP S;  
 1 2 3 4  
 1 6 4 10  
 1 2 -2 4  
 0 0 0 0

Therefore L = and U = .

Their matrix product is ,

so that =

This technique works best when the matrix has fewer rows than columns. If your matrix has fewer columns than rows, find the *LU* decomposition of its *transpose* by the above technique, and take the transpose of the result.

Section 5

**Other Math Pac 2 Improvements**

Complex Square Operation

With the Math Pac 1A, a square operation on a complex number such as Z^2 is internally computed as exp(2\*ln(Z)). With complex numbers, this can limit the accuracy in some cases, such as:

> (1E-9,1)^2  
(-1,2.00000769645E-9)

With the Math Pac 2, the complex square operation Z^2 is now internally computed as Z\*Z to provide the same performance re speed and accuracy for complex numbers as the [x²] key on RPN/RPL calculators (15C, 28C/S, 42S and others).

> (1E-9,1)^2  
(-1,2E-9)

It is still possible to use the general exponentiation formula by forcing a complex power:

> (1E-9,1)^(2,0)  
(-1,2.00000769645E-9)

Moreover, the special case is not based on a pattern detected during the expression parsing but at execution, so an expression like:

X=2 @ Z=(1E-9,1)^X is handled as a multiplication rather than an exponentiation.

Note that the new square functionality operates only on complex numbers, a real square operation like X^2 is still computed by the HP-71 as exp(2\*ln(X)). This generally doesn't affect the accuracy of the result.

A side improvement of the new complex square functionality is that the signed 0 (a specific HP-71 feature) is now conserved by the complex square operation:

> (-0,1)^(2,0)  
(-1,0) the sign of 0 is not preserved with the Math 1A square function 1

> (-0,1)^2  
(-1,-0) the sign of 0 is now preserved

1. This limitation was mentioned by Prof. W. Kahan in his article: "Branch Cuts for Complex Elementary Functions", 1986

RNORM Row and CNORM Column

The RNORM and CNORM functions now store the number of the row (for RNORM) or column (for CNORM) that has the largest sum of absolute values (the norm), these numbers are then available with the new AROW and ACOL functions, until the execution of another function that changes the stored row/column numbers (see section 3).

This functionality is similar to the RNORMROW and CNORMCOL functions of the Series 80.

If more than one row/column (resp. for RNORM/CNORM) has the largest sum of absolute values, the lowest-numbered row/column is chosen and this number is returned as the value of the function AROW/ACOL.

Revised Matrix Algorithms

The original HP-71 Math Pac was sometimes giving results that were slightly different from RPN/RPL calculators for some matrix operations. Parts of the matrix algorithms in the Math Pac 2 have been revised and aligned with the HP-28S and HP-42S calculators.

The main change is related to the internal summation operation that is used in several matrix algorithms.

For instance, computing the determinant of the matrix :  
> OPTION BASE 1  
> DIM A(3,3)  
> MAT INPUT A  
A(1,1)? 69,58,96  
A(2,1)? 51,43,71  
A(3,1)? 32,55,54  
> DET(A)

produced the result 1.00000038313 with the original HP-71 Math Pac 1A,  
but is evaluated as 1.00000038333 on the HP-28S and HP-42S.

The Math Pac 2 matrix algorithms are now aligned with the HP-28S and HP-42S and return the same results as these machines.

Note that this alignment generally doesn't produce more accurate results (the exact determinant of the matrix above is 1), but it makes the HP-71 Math Pac 2 more consistent with the RPN/RPL calculators.

For 2x2 matrices, the DET function now uses a direct and generally more accurate formula to compute the determinant in accordance with the HP-28S and HP-42S calculators.

Shortcuts for INTEGRAL and FNROOT Operations

The Math Pac 2 now offers shortcuts to make the use of the INTEGRAL and FNROOT operations easier and faster especially in direct keyboard calculations:

**Keywords Shortcuts**

INTEGRAL INTEG  
FNROOT FROOT  
IVAR IX  
FVAR FX

The shortcuts can also be used for program line entry and are automatically replaced by the full keyword names.

Examples

Find the value of the integral with an accuracy of 10-5 :

> INTEG(0,1,1E-5,IX^3-IX^2+IX+2)   
 2.41666664282

Find the real root (located between 0 and 1) of the equation :

> FROOT(0,1,7\*FX^3+6\*FX^2+6\*FX-1)  
 .142857142858

Program line entry:

Key in the line:  
 10 X=INTEG(0,1,E,IX^3-IX^2+IX+2) [END LINE]

The line is then displayed as:  
 10 X=INTEGRAL(0,1,E,IVAR^3-IVAR^2+IVAR+2)

Section 6

**Compatibility with Other HP Computers  
and Calculators**

This section summarizes the compatibility of the HP-71 with other HP BASIC computers and with RPN/RPL scientific calculators.

Compatibility with the HP-75 and the Series 80

The Series 70 (HP-71, HP-75) and Series 80 (HP-85, HP-86) share many aspects of their Technical BASIC variants. Thus the compatibility is already very good and is further enhanced by the HP-71 Math Pac 2.

The tables below focus on the array and matrix functions. It is assumed the HP-75 is using the Math Pac ROM and the Series 80 models (noted S80 below) are using the Matrix ROM. The features marked as **X** in bold are provided by the Math Pac 2.

**Scalar real-valued matrix functions**

Keyword 71 75 S80 Comments

DET(A) X X X  
DETL X X X  
FNORM(A) X X X  
CNORM(A) X X X  
CNORMCOL **\*** X \* use ACOL   
RNORM(A) X X X  
RNORMROW **\*** X \* use AROW  
AMIN(A) **X** X X Value of the minimum element  
AMINCOL **\*** X \* use ACOL  
AMINROW **\*** X \* use AROW  
AMAX(A) **X** X X Value of the maximum element  
AMAXCOL **\*** X \* use ACOL  
AMAXROW **\*** X \* use AROW  
MINAB(A) **X** X Value of the smallest el. in abs value  
MAXAB(A) **X** X X Value of the largest el. in abs value  
MAXABCOL **\*** X \* use ACOL  
MAXABROW **\*** X \* use AROW  
SUM(A) **X** X X Sum of the values of all elements  
ABSUM(A) **X** X X Sum of the absolute values of all el.  
DOT(X,Y) X X X  
LBND(A,N) X X X  
UBND(A,N) X X X

**Matrix operations:**

Keyword 71 75 S80 Comments

MAT A=-B X X X  
MAT A=B+C X X X  
MAT A=B-C X X X  
MAT A=B\*C X X X matrix multiply   
MAT A=B.C X element-per-element multiplication  
MAT A=B/C X element-per-element division  
MAT A=(X)\*B X X X multiplication by a scalar  
MAT A=(X)+B X  
MAT A=(X)-B X  
MAT A=(X)/B X  
MAT A=(X)\*B+(Y)\*C X  
MAT A=CROSS(B,C) **X** X X  
MAT A=CSUM(B) **X** X X  
MAT A=RSUM(B) **X** X X  
MAT A=INV(B) X X X  
MAT A=INV(B)\*C **X** X  
MAT A=TRN(B) X X X  
MAT A=TRN(B)\*C X X  
MAT A=B\*TRN(C) X  
MAT X=SYS(A,B) X X X  
MAT A=LUFACT(B) **X** X

**Array input and output:**

Keyword 71 75 S80 Comments

MAT A=B X X X  
MAT A(..)=B(..) X subarray copy  
MAT A=(X) X X X  
MAT A=CON X X X  
MAT A=IDN X X X  
MAT A=ZER X X X  
MAT INPUT A X X X  
MAT READ A \* X X \* use READ A  
MAT DISP [USING] A X X X  
MAT PRINT [USING] A X X X  
REDIM A(..) \* X X \* use DIM A(..)

**Complex matrix operations (HP-71 and HP-75 only):**

Operation 71 75 Comments

det Z=DET(A) MAT Z=CDET(A)  
idn MAT A=IDN MAT A=CIDN  
zer MAT A=ZER MAT A=ZER  
inv MAT A=INV(B) MAT A=CINV(B)  
mult MAT A=B\*C MAT A=CMMULT(B,C)  
sys MAT Z=SYS(A,B) MAT Z=CSYS(A,B)  
conj.trans MAT A=TRN(B) MAT A=CTRN(B)

Compatibility with the Series 200/300 BASIC

The Series 200/300 BASIC (noted RMB below) differs in several aspects from the HP-71 BASIC.

The tables below focus on the array and matrix functions and refer to RMB 5 or later.

**Scalar real- or complex-valued matrix functions:**

RMB Keyword 71 Comments

DET(A) X real and complex matrices  
DET (no operand) X last real matrix determinant   
SUM(A) **X**   
DOT(X,Y) X use DOT2(X,Y) with complex vectors  
BASE(A,N) \* use LBND(A,N)  
SIZE(A,N) \* use UBND(A,N)-LBND(A,N)+1  
RANK(A) \* use 1+(UBND(A,2)>=0)

**Matrix operations:**

RMB operation 71 Comments

MAT A=-B X   
MAT A=B+C X   
MAT A=B-C X   
MAT A=B\*C X matrix multiply   
MAT A=B.C element-per-element multiplication  
MAT A=B/C element-per-element division  
MAT A=B.relop.C element-per-element relation operator  
MAT A=(X)\*B X multiplication by a scalar  
MAT A=(X)+B   
MAT A=(X)-B   
MAT A=(X)/B   
MAT A=(X).relop.B relation operator  
MAT A=CSUM(B) **X**   
MAT A=RSUM(B) **X**   
MAT A=INV(B) X   
MAT A=TRN(B) X   
MAT X=SYS(A,B) X

**Array input and output:**

RMB Keyword 71 Comments

MAT A=B X   
MAT A(..)=B(..) subarray copy  
MAT A=(X) X   
MAT A=IDN X   
READ A(\*) \* use READ A  
INPUT A(\*) \* use MAT INPUT A  
DISP [USING] A(\*) \* use MAT DISP [USING] A  
PRINT [USING] A(\*) \* use MAT PRINT [USING] A  
REDIM A(..) \* use DIM A(..)

Compatibility with RPN/RPL calculators

The HP-71 Math Pac 2 provides several features to increase the functional compatibility with RPN/RPL calculators. However, the names of the corresponding functions may differ between the HP-71 BASIC and RPN/RPL calculator languages. The tables below list the new Math Pac 2 functions and some correspondences between similar functions.

**Complex functions:**

71 28S 42S Comments

(*complex expression*)^2 SQ X^2 complex square  
ASIN/ACOS/ATAN same   
ASINH/ACOSH/ATANH same  
LN same natural logarithm  
EXP EXP E^X  
LOG10 LOG LOG base-10 logarithm   
10^(*complex expression*)ALOG 10^X  
SGN SIGN SIGN

**Array functions and matrix operations:**

71 28S 42S Comments

FNORM ABS FNRM  
CNORM CNRM n/a  
RNORM RNRM RNRM  
DOT/DOT2 DOT DOT See note below for complex vectors   
MAT..CROSS CROSS CROSS 71: real values only  
MAT..RSUM n/a RSUM  
MAT..INV INV INVRT  
MAT..INV()\* \* \* \* 28S/42S: use matrix division  
MAT..SYS \* n/a \* 28S: use matrix division and RSD

Note: the HP-71's DOT function computes the inner product on complex vectors using the complex conjugates of the first vector elements. The HP-28S and HP-42S are using the ordinary product without conjugation. Use DOT2 for full compatibility.

**Appendices**

A. Math Pac 1 and 2 Compatibility

The Math Pac 2 is fully backwards compatible with the original Math Pac.

Programs written for the Math Pac 1A will correctly run with the Math Pac 2 without modification. Some results may be slightly different due to the new improvements provided by the Math Pac 2, see section 5. This applies to:

* the complex square operation Z^2,
* some matrix operations internally using the summation operation: vector dot product, norms, matrix multiplication and inversion, determinant, system solving.

A program using the new Math Pac 2 keywords will not run with the original Math Pac.  
An attempt to execute a program statement that uses new Math Pac 2 functions or MAT operations will produce errors such as:

XFN Not Found or XWORD Not Found

When listing a program with the Math Lex 2 not installed, the new functions will be listed as XFN20xx() and the new MAT operations will be listed as XWORD2064.

Examples:

100 S=SUM(A) will be listed as 100 S=XFN2063(A)  
200 MAT X=CSUM(A) will be listed as 200 XWORD2064

In any case, trying to list or execute a program that uses new Math Pac 2 statements, with Math Pac 1A installed, will neither crash nor corrupt any HP-71 operations.

B. New Memory Requirements

The Math Pac 2 reserves 59.5 bytes of read/write memory for its own uses (The Math Pac 1A was using 43.5 bytes). This section lists the amount of temporary memory used by new Math Pac 2 operations. Refer to the original Math Pac Owner's Manual, Appendix B, for details.

Matrix operations Memory Required For Operation

MAT B=LUFACT(A) Same as MAT B=INV(A)

MAT X=INV(A)\*B Same as MAT X=SYS(A,B) minus 8*NP* bytes if B is real or  
 minus 16*NP* bytes if B is complex

DET(A)/CDET(A) 2*N*(8*N*+1) bytes if A is complex

C. New Error Conditions

The Math Pac 2 adds these new error conditions:

Error  
number Error Message and Condition

1 #Dims

* MAT A=CROSS(B,C) : A or B or C is a matrix
* MAT A=RSUM(B), MAT A=CSUM(B) : A is a matrix or B is a vector
* MAT X=INV(A)\*Y, MAT B=LUFACT(A) : A or B is a vector

2 Not Square

* MAT X=INV(A)\*B, MAT B=LUFACT(A) : A is a not a square matrix

3 Conformability

* MAT X=INV(A)\*B : A and B are not conformable for multiplication
* DOT2(A,B) : A and B don't have the same number of elements

8 Bad Array Size

* MAT A=CROSS(B,C) : the number of elements of vectors B or C is not two or three.

11 ATANH=Inf

* ATANH((1,0)) or ATANH((-1,0))
* ATAN((0,1)) or ATAN((0,-1))

D. New Attention Key Actions

Refer to the original Math Pac Owner's Manual, Appendix D, for details on the [ATTN] key operation.

MAT Statements

The following MAT statements may be halted at any time by pressing [ATTN] *twice*.

MAT *result* = CSUM( *operand* )

MAT *result* = RSUM( *operand* )

MAT *result* = INV( *operand1* ) \* *operand2*

MAT *result* = LUFACT( *operand* )

Scalar-Valued Array Functions

The following scalar-valued functions can be halted at any time by pressing [ATTN] *twice*.

SUM( *operand* )

ABSUM( *operand* )

AMIN( *operand* )

AMAX( *operand* )

MINAB( *operand* )

MAXAB( *operand* )

CDET( *operand* )

DOT2( *operand* )

For these functions, the HP-71 will display the error message Function Interrupted.

**Keyword Index**

Page numbers starting with M refer to the original Math Pac Owner's Manual.  
Page numbers **in bold** starting with **S** refer to this Math Pac 2 Manual Supplement.

**Keyword Page Description**

ABS M41 Absolute value of a complex number.  
ABSUM **S15** Array element absolute value sum.  
ACOL **S17** Array Column.  
ACOS **S10** Complex inverse cosine.  
ACOSH M28 Inverse hyperbolic cosine.  
ACOSH **S9** Complex inverse hyperbolic cosine.  
ACS **S10** Same as ACOS.  
AMAX **S16** Array element maximum.  
AMIN **S16** Array element minimum.  
ARG M41 Argument of a complex number.  
AROW **S17** Array row.  
ASIN **S10** Complex inverse sine.  
ASINH M28 Inverse hyperbolic sine.  
ASINH **S9** Complex inverse hyperbolic sine.  
ASN **S10** Same as ASIN.  
ATAN **S10** Complex inverse tangent.  
ATANH M28 Inverse hyperbolic tangent.  
ATANH **S9** Complex inverse hyperbolic tangent.  
ATN **S10** Same as ATAN.  
BSTR$ M16 Decimal to binary/octal/hexadecimal conversion.  
BVAL M15 Binary/octal/hexadecimal to decimal conversion.  
C(,) M22 Complex IMAGE field.  
CDET **S20** Determinant of a complex matrix, same as DET.  
CNORM M70 One-norm (column norm) of an array.  
COMPLEX M19 Complex variable creation.  
COMPLEX SHORT M19 Complex short variable creation.  
(,) M21 Conversion, real to complex.  
CONJ M42 Complex conjugate.  
COS M38 Complex cosine.  
COSH M27 Hyperbolic cosine.  
COSH M39 Complex hyperbolic cosine.  
COT **S8** Cotangent function.  
CSC **S8** Cosecant function.  
DET M69 Determinant of a matrix.  
DET **S20** Determinant of a complex matrix.  
  
DET (no operand) M69 Determinant of last real matrix used as operand of INV or  
 LUFACT, or first operand of INV()\* or SYS.  
DETL M69 Same as DET (no operand).  
DOT M71 Dot (inner) product.  
DOT2 **S21** Alternate dot (inner) product.  
EXP M37 Complex exponential (*eZ*).  
FGUESS M90 Second-best guess to value returned by last FNROOT.  
FNORM M70 Frobenius norm.  
FNROOT M89 Rootfinding for functions.  
FROOT **S31** Shortcut for FNROOT.  
FVALUE M90 Functional value of last FNROOT.  
FVAR M90 Variable to solve for in FNROOT.  
FX **S31** Shortcut for FVAR.  
GAMMA M28 Gamma function.  
IBOUND M103 Uncertainty of last INTEGRAL.  
IMPT M21 Imaginary part of complex number.  
INTEGRAL M101 Integration of functions.  
INTEG **S31** Shortcut for INTEGRAL.  
IROUND M30 Integer round.  
IVALUE M102 Current approximation to an INTEGRAL.  
IVAR M102 Variable of integration in INTEGRAL.  
IX **S31** Shortcut for IVAR.  
LBND M72 Array subscript lower bound.  
LBOUND M71 Same as LBND.  
LGT **S11** Complex base-10 logarithm.  
LOG M37 Complex natural logarithm.  
LOG10 **S11** Same as LGT.  
LOG2 M29 Log base 2.  
MAT DISP M54 Array display (unformatted).  
MAT DISP USING M55 Array display (formatted).  
MAT INPUT M53 Interactive array input.  
MAT PRINT M55 Array printing (unformatted).  
MAT PRINT USING M56 Array printing (formatted).  
MAT .. = M51 Array copying (simple assignment).  
MAT .. + M64 Array addition.  
MAT .. - M63 Array negation.  
MAT .. - M64 Array subtraction.  
MAT .. \* M65 Matrix multiplication.  
MAT .. () M52 Scalar to array assignment (numeric expression assignment).  
MAT .. ()\* M65 Scalar multiplication.  
MAT .. CON M52 Constant array with optional redimensioning.  
MAT .. CROSS **S22** Vector cross product.  
MAT .. CSUM **S23** Array column sum.  
MAT .. FOUR M135 Finite Fourier transform.  
MAT .. IDN M52 Identity array with optional redimensioning.  
MAT .. INV M77 Matrix inversion.  
MAT .. INV()\* **S24** Matrix division.  
MAT .. LUFACT **S26** Matrix *LU* decomposition.  
MAT .. PROOT M120 Polynomial rootfinding.  
MAT .. RSUM **S23** Array row sum.  
MAT .. SYS M79 System solution.  
MAT .. TRN M77 Transpose or conjugate transpose.  
MAT .. TRN()\* M66 Transpose or conjugate transpose multiply.  
MAT .. ZER M53 Zero array with optional redimensioning.  
MAT .. ZERO M53 Same as MAT..ZER.  
MAXAB **S16** Array element maximum absolute value.  
MINAB **S16** Array element minimum absolute value.  
NAN$ M30 NaN diagnostic function.  
NEIGHBOR M30 Successor/predecessor function.  
POLAR M40 Rectangular to polar function.  
PROJ M42 Conversion of complex infinities to projective infinities.  
RECT M40 Polar to rectangular function.  
RNORM M70 Infinity (row) norm of an array.  
SCALE10 M29 Exponent scaling function.  
SEC **S8** Secant function.  
SGN M41 Complex unit vector.  
SIN M38 Complex sine.  
SINH M27 Hyperbolic sine.  
SINH M39 Complex hyperbolic sine.  
SQR M40 Complex square root.  
SQRT M40 Same as SQR.  
SUM **S15** Array element sum.  
TAN M38 Complex tangent.  
TANH M27 Hyperbolic tangent.  
TANH M39 Complex hyperbolic tangent.  
TYPE M31 Data type function.  
UBND M71 Array subscript upper bound.  
UBOUND M71 Same as UBND.  
+ M35 Complex addition.  
- M35 Complex unary minus.  
- M36 Complex subtraction.  
\* M36 Complex multiplication.  
/ M36 Complex division.  
^ M36 Complex exponentiation (*ZW*).  
= < > # ? M43 Complex relational operators.